Journal of Engineering Physics and Thermophysics, Vol. 71, No. 5, 1998

USE OF THE INTEGRO-INTERPOLATION METHOD FOR CONSTRUCTION OF DIFFERENCE EQUATIONS FOR DETERMINATION OF THERMAL PROPERTIES AND UNSTEADY-STATE HEAT FLUXES

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UDC 681.2:536.083

Theoretical methods are considered for determination of temperature-dependent thermal properties and unsteady-state heat fluxes. The methods are based on difference equations of heat conduction obtained by the integro-interpolation method for plane, cylindrical, and spherical variants of one-dimensional temperature fields.

1. Methods of Measurement of Unsteady-State Heat Fluxes and Thermal Properties. In thermophysical research or in studies of unsteady-state heat transfer, use is made of methods based on numerical solution of the inverse heat-conduction problem, for example, [1, 2]. It is obvious that they are quite efficient but too complicated and unwieldy to be used in large-scale automated measurements.

For measurement of steady-state and slowly changing heat fluxes calorimeters are used whose principle of operation is based on interpolation of the Fourier equation

$$q = \lambda \nabla t_x = \lambda \, (\Delta t / \Delta x) \, . \tag{1}$$

However, in measurement of unsteady-state heat fluxes, they give a large error. For example, in the model case in which the calorimeter has the characteristics $\lambda = 0.32$ W/(m·K), $C = 2 \cdot 10^3$ kJ/(m^3·K), $x = 10^{-3}$ m and a temperature jump occurred at one of the boundaries, while the other is in ideal thermal contact with a semi-infinite body having similar thermal properties (TP), the methodological error in determining the heat flux at the boundaries of the calorimeter changes from 50% to 4% in the first 25 hours [3].

Calorimeters that take into consideration the rate of change of their volume-average temperature provide higher accuracy of measurement of unsteady-state heat flux densities. In [4] a version of such calorimeters is considered. It is made of metal and its mathematical formula is obtained for quasisteady conditions. Its design is rather complicated and, as a result, it gives a methodological error in the case of an unsteady-state rate of change of its temperature.

Analytical relations of one-dimensional plane, cylindrical, and spherical temperature fields in objects of measurements are a theoretical basis for most modern methods of measurement of TP [4]. Therefore, availability of an analytical solution and rather simple and accurate realization of the conditions specified by the boundary-value of heat conduction problem are a criterion for choosing a mathematical model and a thermal action that can be a basis for a method of measuring TP. Such methods are considered in detail in [4].

When this approach to designing methods of measurement of TP is used, the following problems are inevitable. First, it is necessary to provide accurately initial and boundary conditions that strictly correspond to the chosen thermal model. Second, the obtained analytical solutions are unwieldy, and some difficulties can arise in their truncation and in expression of the sought quantity in an explicit form. Moreover, in determination of the temperature dependence of the thermal conductivity and the heat capacity, for example; in the form of a polynomial representation of these functions the problem has no exact solution.

Novomoskovsk Institute of the D. I. Mendeleev Russian Chemical Engineering University, Novomoskovsk, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 71, No. 5, pp. 811-818, September-October, 1998. Original article submitted March 25, 1997.

2. Mathematical Formulation of the Measuring Problem. The object of study or a calorimeter that is subjected to a thermal action and the temperature field in it are described by a one-dimensional equation of the form

$$\nabla \left\{x^{\alpha} \lambda(t) \nabla t_x(x, \tau)\right\}_x = x^{\alpha} C(t) \nabla t_{\tau}(x, \tau), \quad x \in [x_0, x_n], \quad \tau \in [\tau_0, \tau_k]. \tag{2}$$

The temperature at the boundaries of the object of study (the calorimeter) and at some points is known, i.e.,

$$t(x_i, \tau) = T_i(\tau), \quad x_0 \le x_i \le x_n \quad (i = 0, 1, ..., n). \tag{3}$$

It is necessary to determine: (a) the heat flux density at the boundaries of the calorimeter $q(x_0, \tau)$, $\{q(x_n, \tau)\}$ or the quantity of heat $Q(x_0, \tau_{j+1} - \tau_j) = \int_{\tau_j}^{\tau_{j+1}} q(x_0, \tau) d\tau$, $\{Q(x_n, \tau_{j+1} - \tau_j) = \int_{\tau_j}^{\tau_{j+1}} q(x_n, \tau) d\tau\}$ from the known $C(t) = C_0$ and $\lambda(t) = \lambda_0$; (b) the temperature dependence of the thermal conductivity $\lambda(t) = \sum_0^U \lambda_u t^u$ and the heat capacity per unit volume $C(t) = \sum_0^V C_v t^v$ from the known $Q(x_0, \tau) = Q_0(\tau)$, $\{Q(x_n, \tau) = Q_n(\tau)\}$.

3. Theoretical Bases for Obtaining Equations for Determination of the Heat Flux Density and Thermal Properties. We consider an integro-interpolation method (IIM) of obtaining difference equations that is based on double integration of Eq. (2) with respect to the coordinate x [5]. It should be noted that the idea of double integration of a differential equation is used in Streitz's method [6] for identification of objects with lumped parameters. Equation (2) will be integrated from x_i to x:

$$\lambda(t) \left[\nabla t_x(x, \tau) - x_i^{\alpha} x^{-\alpha} \nabla t_x(x_i, \tau)\right] = x^{-\alpha} \int_{x_i}^{x} C(t) x^{\alpha} \nabla t_{\tau}(x, \tau) dx. \tag{4}$$

Then, this identity is integrated with respect to x from x_i to x_n and from x_0 to x_i:

$$L_{\alpha} q(x_i, \tau) = \int_{x_i}^{x_n} \lambda(t) \nabla t_x(x, \tau) dx + \int_{x_i}^{x_n} x^{-\alpha} dx \int_{x_i}^{x} C(t) x^{\alpha} \nabla t_{\tau}(\xi, \tau) d\xi, \tag{5}$$

$$L_{\alpha}^{*} q(x_i, \tau) = \int_{x_0}^{x_i} \lambda(t) \nabla t_x(x, \tau) dx + \int_{x_0}^{x_i} x^{-\alpha} dx \int_{x_i}^{x} C(t) x^{\alpha} \nabla t_{\tau}(\xi, \tau) d\xi. \tag{6}$$

With the order of integration changed, the second terms in Eqs. (5) and (6) are transformed:

$$\int_{x_i}^{x_n} x^{-\alpha} dx \int_{x_i}^{x} C(t) x^{\alpha} \nabla t_{\tau}(\xi, \tau) d\xi = \int_{x_i}^{x_n} C(t) (x_n - \xi) \nabla t_{\tau}(\xi, \tau) d\xi, \tag{7}$$

$$\int_{x_0}^{x_i} x^{-\alpha} dx \int_{x_i}^{x} C(t) x^{\alpha} \nabla t_{\tau}(\xi, \tau) d\xi = -\int_{x_0}^{x_i} C(t) (\xi - x_0) \nabla t_{\tau}(\xi, \tau) d\xi. \tag{8}$$

With account for the temperature dependences $\lambda(t)$ and $C(t)$ and transformations (7) and (8), Eqs. (5) and (6) take the form

$$L_\alpha q(x_i,\tau) = \sum_{u=0}^{U} \frac{\lambda_u}{u+1}\left[t^{u+1}(x_i,\tau) - t^{u+1}(x_n,\tau)\right] +$$

$$+ \sum_{\nu=0}^{V} \frac{C_\nu}{\nu+1} \int_{x_i}^{x_n} (x_n - \xi)\,\nabla\left\{t^{\nu+1}(\xi,\tau)\right\}_\tau d\xi\,, \tag{9}$$

$$L^*_\alpha q(x_i,\tau) = \sum_{u=0}^{U} \frac{\lambda_u}{u+1}\left[t^{u+1}(x_0,\tau) - t^{u+1}(x_i,\tau)\right] -$$

$$- \sum_{\nu=0}^{V} \frac{C_\nu}{\nu+1} \int_{x_0}^{x_i} (\xi - x_0)\,\nabla\left\{t^{\nu+1}(\xi,\tau)\right\}_\tau d\xi\,. \tag{10}$$

The function $t^{\nu+1}(x)$ $(\nu = 0, 1, \ldots, V)$ is replaced by the Lagrange interpolation polynomial of n-th degree that coincides with it at the points x_i $(i = 0, \ldots, n)$:

$$t^{\nu+1}(x) = P_n(x) = \sum_{i=0}^{n} Q_n^{(i)}(x)\,t^{\nu+1}(x_i) \quad (\nu = 0, 1, \ldots, V)\,, \tag{11}$$

where $Q_n^{(i)}(x)$ are polynomials of n-th degree defined by the equalities [7]

$$Q_n^{(i)}(x) = \frac{(x - x_0)\ldots(x - x_{i-1})(x - x_{i+1})\ldots(x - x_n)}{(x_i - x_0)\ldots(x_i - x_{i-1})(x_i - x_{i+1})\ldots(x_i - x_n)} \quad (i = 0, 1, \ldots, n)\,. \tag{12}$$

Use of the Lagrange interpolation polynomial gives quadrature formulas that are exact for polynomials of n-th degree. As a result, Eqs. (7) and (8) are rewritten in the form

$$\int_{x_i}^{x_n} C(t)(x_n - \xi)\,\nabla\left\{t^{\nu+1}(\xi,\tau)\right\}_\tau d\xi = \sum_{\nu=0}^{V} \frac{C_\nu}{\nu+1}\nabla\left\{\sum_{i=0}^{n} t^{\nu+1}(x_i,\tau)\right\}_\tau \int_{x_i}^{x_n}(x_n - \xi)\times$$

$$\times Q_n^{(i)}(\xi)\,d\xi = \sum_{\nu=0}^{V} \frac{C_\nu}{\nu+1}\nabla\left\{\sum_{i=0}^{n} p_i\,t^{\nu+1}(x_i,\tau)\right\}_\tau\,, \tag{13}$$

$$\int_{x_0}^{x_i} C(t)(\xi - x_0)\,\nabla\left\{t^{\nu+1}(\xi,\tau)\right\}_\tau d\xi = \sum_{\nu=0}^{V} \frac{C_\nu}{\nu+1}\nabla\left\{\sum_{i=0}^{n} t^{\nu+1}(x_i,\tau)\right\}_\tau \int_{x_0}^{x_n}(\xi - x_0)\times$$

$$\times Q_n^{(i)}(\xi)\,d\xi = \sum_{\nu=0}^{V} \frac{C_\nu}{\nu+1}\nabla\left\{\sum_{i=0}^{n} p_i^*\,t^{\nu+1}(x_i,\tau)\right\}_\tau\,. \tag{14}$$

Substitution of (13) and (14) into (9) and (10) gives

$$L_\alpha q(x_i,\tau) = \sum_{u=0}^{U} \frac{\lambda_u}{u+1}\left[t^{u+1}(x_i,\tau) - t^{u+1}(x_n,\tau)\right] + \sum_{\nu=0}^{V} \frac{C_\nu}{\nu+1}\sum_{i=0}^{n} p_i\nabla\left\{t^{\nu+1}(x_i,\tau)\right\}_\tau\,, \tag{15}$$

$$L^*_\alpha q(x_i,\tau) = \sum_{u=0}^{U} \frac{\lambda_u}{u+1}\left[t^{u+1}(x_0,\tau) - t^{u+1}(x_i,\tau)\right] + \sum_{\nu=0}^{V} \frac{C_\nu}{\nu+1}\sum_{i=0}^{n} p_i^*\nabla\left\{t^{\nu+1}(x_i,\tau)\right\}_\tau\,, \tag{16}$$

which are a basis for designing methods of measurement of TP and unsteady-state heat fluxes or quantities of heat.

We give formulas for determination of the characteristic dimension L_α for plane, cylindrical, and spherical temperature fields:

$$L_0 = L_0^* = x_n - x_0\,;\quad L_1 = x_0 \ln\left(\frac{x_n}{x_0}\right)\,;\quad L_1^* = x_n \ln\left(\frac{x_n}{x_0}\right)\,;$$

$$L_2 = \frac{x_0}{x_n}(x_n - x_0)\,;\quad L_2^* = \frac{x_n}{x_0}(x_n - x_0)\,.$$

4. Obtaining Mathematical Formulas for Determination of Unsteady-State Heat Flux Densities (Quantities of Heat) at the Boundaries of the Calorimeter and TP of the Object of Study. In order to determine the heat flux density (the quantity of heat in the time $\tau_{j+1} - \tau_j$) at the boundaries of the calorimeter (i.e., at the points x_0 and x_n) use is made of Eqs. (15) and (16) and information specified by the conditions of the measuring problem. As a result, we have the following mathematical formulas:

$$q(x_0, \tau) = \frac{\lambda_0}{L_\alpha}[T_0(\tau) - T_n(\tau)] + \frac{C_0}{L_\alpha}\sum_{i=0}^{n} p_i \nabla T_{\tau,i}(\tau)\,, \tag{17}$$

$$q(x_n, \tau) = \frac{\lambda_0}{L_\alpha}[T_0(\tau) - T_n(\tau)] - \frac{C_0}{L_\alpha}\sum_{i=0}^{n} p_i^* \nabla T_{\tau,i}(\tau)\,, \tag{18}$$

$$Q(x_0, \tau_{j+1} - \tau_j) = \frac{\lambda_0}{L_\alpha}\int_{\tau_j}^{\tau_{j+1}}[T_0(\tau) - T_n(\tau)]\,d\tau + \frac{C_0}{L_\alpha}\sum_{i=0}^{n} p_i T_i(\tau_{j+1} - \tau_j)\,, \tag{19}$$

$$Q(x_n, \tau_{j+1} - \tau_j) = \frac{\lambda_0}{L_\alpha}\int_{\tau_j}^{\tau_{j+1}}[T_0(\tau) - T_n(\tau)]\,d\tau - \frac{C_0}{L_\alpha}\sum_{i=0}^{n} p_i^* T_i(\tau_{j+1} - \tau_j)\,. \tag{20}$$

For their simplification and concretization, we take only three points of temperature measurement. We choose $x_1 = 0.5(x_2 + x_0)$. After integration of Eqs. (15) and (16) with respect to time with the limits $[\tau_j, \tau_{j+1}]$, where $j = 0, 1, \ldots, k$, with account for information specified in the measuring problem, a system of algebraic equations is obtained:

$$L_\alpha Q_0(\tau_{j+1} - \tau_j) = \sum_{u=0}^{U}\frac{\lambda_u}{u+1}\int_{\tau_j}^{\tau_{j+1}}[T_0^{u+1}(\tau) - T_2^{u+1}(\tau)]\,d\tau + \sum_{v=0}^{V}\frac{C_v}{v+1}\sum_{i=0}^{i=2} p_i T_i^{v+1}(\tau_{j+1} - \tau_j)\,, \tag{21}$$

$$L_\alpha Q(x_1, \tau_{j+1} - \tau_j) = \sum_{u=0}^{U}\frac{\lambda_u}{u+1}\int_{\tau_j}^{\tau_{j+1}}[T_1^{u+1}(\tau) - T_2^{u+1}(\tau)]\,d\tau + \sum_{v=0}^{V}\frac{C_v}{v+1}\sum_{i=1}^{i=2} p_i T_i^{v+1}(\tau_{j+1} - \tau_j)\,, \tag{22}$$

$$L_\alpha^* Q(x_1, \tau_{j+1} - \tau_j) = \sum_{u=0}^{U}\frac{\lambda_u}{u+1}\int_{\tau_j}^{\tau_{j+1}}[T_0^{u+1}(\tau) - T_1^{u+1}(\tau)]\,d\tau -$$

$$- \sum_{v=0}^{V}\frac{C_v}{v+1}\sum_{i=0}^{i=1} p_i^* T_i^{v+1}(\tau_{j+1} - \tau_j) \quad (j = 0, 1, \ldots, k)\,. \tag{23}$$

The value of k is chosen proceeding from the number of coefficients λ_u and C_v to be determined and the number of chosen equations: (21) or (21), (22), and (23).

We give formulas for determination of TP obtained from Eqs. (21), (22), and (23) for the case in which $\lambda(t) = \lambda_0$ and $C(t) = C_0$:

$$a_0 = \frac{\lambda_0}{C_0} = l^2 \frac{T_0(\tau_1 - \tau_0) + 4T_1(\tau_1 - \tau_0) + T_2(\tau_1 - \tau_0)}{6 \int\limits_{\tau_0}^{\tau_1} [T_0(\tau) - 2T_1(\tau) + T_2(\tau)]\, d\tau}. \tag{24}$$

The solution of the system of algebraic equations

$$Q_0(\tau_{j+1} - \tau_j) = \frac{\lambda_0}{L_\alpha} \int\limits_{\tau_j}^{\tau_{j+1}} [T_0(\tau) - T_1(\tau)]\, d\tau + \frac{C_0}{L_\alpha} \sum_{i=0}^{i=2} p_i T_i(\tau_{j+1} - \tau_j) \quad (j = 0, 1) \tag{25}$$

gives λ_0 and C_0 [8]. If the condition

$$t(x, \tau_{j+1}) = t(x, \tau_j) \tag{26}$$

is provided in the measurement, the mathematical formula for determination of the thermal conductivity has the form

$$\lambda_0 = L_\alpha \frac{Q_0(\tau_{j+1} - \tau_j)}{\int\limits_{\tau_j}^{\tau_{j+1}} [T_0(\tau) - T_2(\tau)]\, d\tau}. \tag{27}$$

The initial temperatures of the object of study entering into Eqs. (24), (25), and (27) will automatically be taken into consideration with their different values in the measuring process. Therefore, if a TP is considered to be a constant quantity within a rather narrow temperature range within which the temperature varies in the process of measurement, these equations can be used for determination of the TP for different initial temperatures in the range of study.

5. Sources of Errors. All errors of measurement of TP and heat flux densities (quantities of heat) can be divided into three groups: 1) errors caused by the contact method of temperature measurement; 2) errors caused by inaccurate determination of the constant coefficients in the calculational formulas; (3) methodological errors from the inadequacy of the chosen model, the inexactness of satisfaction of the initial and boundary conditions, and the use of approximate formulas.

Errors of groups 1 and 2 are not considered in this article since they are largely general for any method of measurement of TP and heat fluxes. We consider the components of the methodological error.

When we obtained the difference equations, we used approximate formulas (13) and (14) for determination of an integral parameter over the coordinate x. This error will be evaluated by the method of obtaining an exact expression for the approximation of a quadrature formula [7]. As a result, the following relation is obtained:

$$\Delta = \int\limits_{x_0}^{x_n} (x_n - \xi)\, t(\xi)\, d\xi - \sum_{i=0}^{n} p_i\, t(x_i) = \int\limits_{x_0}^{x_n} F_r(\xi)\, t^{(r)}(\xi)\, (x_n - \xi)\, d\xi, \tag{28}$$

where

$$F_r(\xi) = \frac{1}{(r-1)!} \left[\frac{(x_n - \xi)^r}{r} - \sum_{i=0}^{n} p_i K_r(x_i - \xi) \right], \tag{29}$$

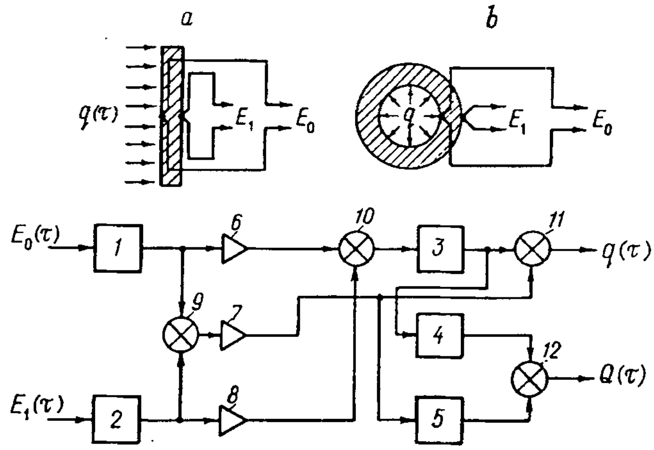

Fig. 1. Measurement of heat flux density: a, b) schemes of calorimeters for plane and cylindrical temperature fields; c) a device for automatic measurement of heat flux density and quantity of heat.

$$K_r(x_i - \xi) = \begin{cases} (x_i - \xi)^{(r-1)} & x_i - \xi \geq 0, \\ 0 & x_i - \xi < 0. \end{cases} \tag{30}$$

In final form the evaluation of the approximation can be expressed as follows:

$$\Delta \leq M \int_{x_0}^{x_n} |F_r(\xi)(x_i - \xi)| \, d\xi = MS_r. \tag{31}$$

For $x_0 = 0$ and $x_2 = L$ with $P_1(x) = a_0 + a_1 x$, we obtain $S_0 = 0.252L^2$, $S_1 = 0.070L^2$.

The quantity M is expressed in terms of the maximum heat flux in the calorimeter or in terms of the maximum rate of change of the temperature in the object:

$$M_1 = \frac{q_{\max}}{\lambda_0}, \quad M_2 = \frac{1}{a_0}\left[\max \frac{dt}{d\tau}\right]. \tag{32}$$

Thus, proceeding from a priori information about the maximum rate of change of the measured heat flux, it is possible to evaluate the methodological error caused by approximate formulas (13) and (14) for determination of an integral parameter over the coordinate x. In the case of measurement of TP and quantities of heat, this error is evaluated in terms of the maximum rate of change of the temperature of the object at the times τ_j ($j = 0, ..., k$) of the beginning and end of integration. On the other hand, with this evaluation it is possible to establish heating conditions under which the error would not exceed a specified value.

A second component of the error in comprehensive determination of TP is caused by the instability of the solution of the system of algebraic equations (21), which is a consequence of the ill-posedness of the inverse heat-conduction problem. This component can be diminished by simultaneous use of Eqs. (21), (22), and (23) or by taking measurements in heating and cooling stages. For example, in the case of the system of equations (21) for $V = U = 1$, two heating stages $[\tau_0, \tau_1]$, $[\tau_1, \tau_2]$ and two cooling stages $[\tau_2, \tau_3]$, $[\tau_3, \tau_4]$ can be used.

6. Technical Implementation, Application, and Discussion of Obtained Solutions. In measurement of unsteady-state heat flux densities (or quantities of heat) the plane and cylindrical versions of the method are the most useful. In the second case heat fluxes from liquid and gaseous heat transfer agents in tubes can be measured. Schemes of calorimeters and a device for continuous and automatic determination of the parameters mentioned are given in Fig. 1. Depending of the source of the heat flux, the calorimeter can be a thin plate or a hollow cylinder at whose boundaries temperatures are measured. The accuracy of determination of the heat flux density can be increased by using intermediate points x_i of temperature measurement. The automatic device contains two thermal-

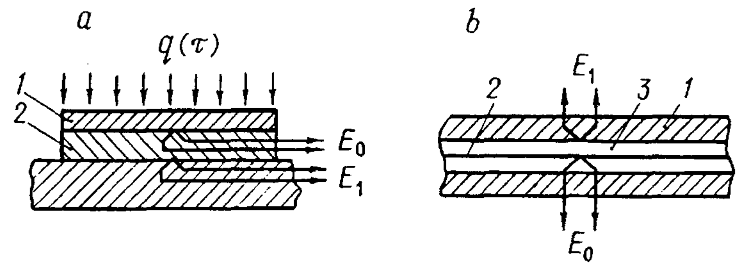

Fig. 2. Schemes of thermal measuring units for determination of thermal properties of: a) solid materials; b) liquids and gases.

emf amplifiers 1 and 2, differentiator 3, integrators 4 and 5, and adders 9, 10, 11, and 12 with coefficient setters 6, 7, and 8. This device implements fromulas (17) and (19).

Examples of using the obtained equations for determination of TP are shown in Fig. 2. In the first case (Fig. 2a) a flat specimen is heated by an electric heater; temperatures are measured at its boundaries, and calorimeter 1 measures the quantity of heat supplied to specimen 2. The thermal conductivity and heat capacity can be determined with the device suggested in [9]. When only the thermal conductivity is measured, the device implements the simpler formula [10]

$$\lambda = \frac{\lambda_c h_{sp} \int_{\tau_j}^{\tau_{j+1}} \Delta T_c(\tau)\, d\tau}{h_c \int_{\tau_j}^{\tau_{j+1}} \Delta T_{sp}(\tau)} . \qquad (33)$$

For measurement of TP of liquids and gases, an object (Fig. 2b) in the form of a cylinder with a small diameter 1 filled with the studied material 3 can be conveniently used. In this case the heater is a wire 2 located on the axis of the cylinder, and the temperature of the heater and the inner surface of the cylinder is measured. The quantity of heat is determined from the power of the heater and the time of heating.

A thermal-conductivity meter for high-conductivity materials is widely used at temperatures from room temperature to 150°C [11]. More than 20 instruments of various modifications (in particular, IT-02Ts) have been manufactured for various organizations that develop and produce high-conductivity metal ceramics used in the electronics industry. The thermal-conductivity meter has metrological provision that includes a checking procedure and thermal-conductivity standards made of steel 12Kh18N10T, low-carbon steel, molybdenum MVCh, and copper M1 manufactured and certified at the Dal'standart R&P Corporation. The measurement error is within 7%.

It seems useful to enumerate some important advantages of the obtained solutions of the inverse heat-conduction problem: 1) an analytical solution of the direct heat-conduction problem is unnecessary and, consequently, there are no problems connected with this solution (specification of initial conditions, maintaining the boundary conditions with high accuracy, limitation of the number of terms in the functional series representing the analytical solution, and expression of the sought quantity in explicit form); 2) it is possible to evaluate all the components of the error of measurement of TP and heat flux densities; 3) a wide choice of heating conditions for the object of study is ensured so as to reduce the time of measurement and the error; 4) an exact mathematical formula is derived for determination of thermal conductivity; 5) the equations allow determination of the coefficients in the polynomial dependence of thermal conductivity and heat capacity per unit volume on temperature. Moreover, it is possible to the measure thermal conductivity $\lambda(t_\xi)$ and heat capacity per unit volume $C(t_\xi)$ for different initial temperatures in the specified range.

The solutions obtained allow development of effective methods of measurement of thermal properties and heat flux densities that can be widely used in practical measurements of thermal properties.

NOTATION

Δt, temperature across a calorimeter of thickness Δx; $\alpha = 0, 1, 2$, power corresponding to plane, cylindrical, and spherical temperature fields; $\lambda(t)$, $C(t)$, thermal conductivity and heat capacity per unit volume of the object of study or the calorimeter; $L_\alpha = x_i^\alpha \int_{x_i}^{x_n} x^{-\alpha}dx$, $L_\alpha^* = x_i^\alpha \int_{x_0}^{x_i} x^{-\alpha}dx$, characteristic dimensions corresponding to different variants of the temperature fields; ∇t_x, ∇t_τ, derivatives of the function $t(x, \tau)$ with respect to x and τ, respectively; $\nabla\{\cdot\}_\tau$, time derivative of the variable $\{\cdot\}$; $p_i = \int_{x_i}^{x_n} (x_n - \varepsilon)Q_n^{(i)}(\xi)d\xi$, $p_i^* = \int_{x_0}^{x_i} (\xi - x_0)Q_n^{(i)}(\xi)d\xi$, weight coefficients; $\nabla T_{\tau,i}$, $T_i(\tau_{j+1} - \tau_j)$, rate of change of the temperature with time at the point x_i and temperature increment at the point x_i in the time $\tau_{j+1} - \tau_j$, respectively; $l = 0.5(x_2 - x_0)$; r, natural number corresponding to the order of the derivative; M, maximum value of the derivative of order r, i.e., max $t_x^{(r)}$; $q_{\max}$, maximum heat flux density; h_{sp}, h_{c}, thickness of the specimen and the calorimeter; λ_{c}, thermal conductivity of the material of the calorimeter; $\Delta T_{\text{c}}(\tau)$, $\Delta T_{\text{sp}}(\tau)$, temperature difference across the calorimeter and the specimen; a_0, a_1, coefficients of the interpolation polynomial of first degree.

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